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LAB PARTNERS: _____

FORCE TABLE LAB 1

1 Introduction

The idea of vectors is one of the most fundamental and useful in all of physics. There are many different quantities that can be expressed as a vector, including the force acting on a particle. In this experiment we will review and apply the main ideas of vectors, including their addition, both graphically and algebraically. After reviewing the basic ideas of vectors, we will use a force table to understand how to add together the different forces acting on a system to determine the conditions for equilibrium. The system is in equilibrium when the sum of the forces acting on that system is zero.

2 What Are Vectors?

Some properties of a system can be entirely described in terms of a single number. Once this number is given, we know everything about that property of the system. For example, a rock might have a mass of 500 grams, or the temperature of a room might be 23° C. Quantities that can be specified in terms of a single number are called *scalars*. Examples of scalars include mass and temperature, as well as other quantities like energy, or electric charge.

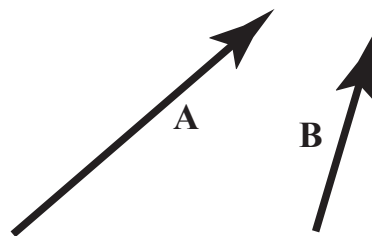
Then there are other quantities that need more information to be completely specified. Suppose you want to take a drive along the freeway at 70 miles per hour. You've decided on a speed, but that's not quite enough to figure out where you're going. You also need a *direction* to drive, say north. Once you have a speed and a direction, only then do you know enough to start your drive.

Your speed and direction together give you your *velocity*. To fully determine the velocity we had to specify a *magnitude* (the speed, 70 mph) and also a *direction* (north). This is different than scalars, which had only a magnitude associated with them; for example, the mass of the rock was 500 grams – no direction needed. Quantities that have both a magnitude and direction are called *vectors*. Examples of vectors include velocity, acceleration, momentum and, as we'll use below, force.

Now that we have an idea of what vectors are, let's discuss some ways of representing them in calculations.

2.1 Vectors as Arrows

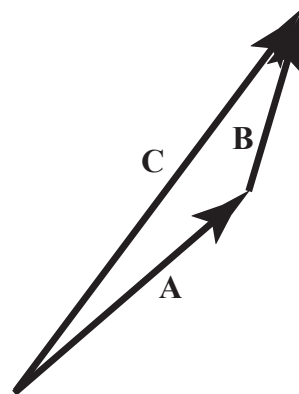
Because a vector has a magnitude and direction associated with it, we can easily picture it as an arrow, as seen in the figure to the right. We can label this vector as **A** (often times the vector is instead labeled by an arrow over it, \vec{A} , especially when written by hand). The arrow has a magnitude (it's length), and also points in a certain direction, and so it contains everything we need to describe the vector. A different vector, **B**, points in a different direction, and has a different length, and so a different magnitude.



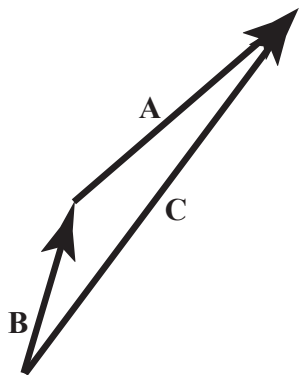
2.1.1 Adding the Arrows

Suppose we want to *add* together the arrows, **A** and **B**. Why might we want to do this? Suppose the vectors **A** and **B** represent displacements; we walk 1.5 km to the northeast (**A**), and then turn a bit more north and walk another 1 km (**B**). How far have we gone from our starting place? We can figure this out by adding together the two vectors. Adding together the vectors is as simple as placing the tail of **B** on the head of **A**. Then you draw a new vector **C** from the tail of **A** to the head of **B**. The resulting vector **C** is the sum of **A** and **B**,

$$\mathbf{C} = \mathbf{A} + \mathbf{B}.$$



We can always slide the arrows around, keeping the length the same, *as long as we don't change the angles!* So, we are always free to put the tail of one vector on the head of the other.



Suppose that we want to add the vectors in the *opposite* order. In our earlier example, we instead walk mostly north for 1 km, and then turn toward the east and walk for 1.5 km. How far do we go? Do we end up in the same place? In order to add the vectors in the opposite order we start with the vector **B**, and place the tail of **A** on the tip of **B**. We then draw the resulting vector **C**, as in the figure to the left, and so

$$\mathbf{C} = \mathbf{B} + \mathbf{A}.$$

The resultant vector, \mathbf{C} , that we get adding \mathbf{A} and \mathbf{B} in different orders looks the same, but we can check that it really is the same. Consider the diagram to the left, where \mathbf{A} and \mathbf{B} are added in different orders, combining the previous two figures. It's clear that the resultant vector is the same in both cases. So, it doesn't matter in which order you add the vectors,

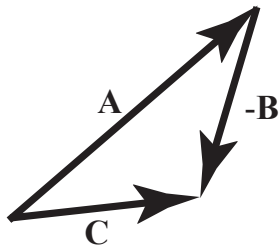
$$\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}.$$

The addition of vectors is *commutative*.

Suppose that we have *three* vectors, \mathbf{A} , \mathbf{B} , and \mathbf{C} . We can add all three of these vectors as before, first putting the tail of \mathbf{B} on the head of \mathbf{A} , then placing the tail of \mathbf{C} on the head of \mathbf{B} . Then you draw the resultant vector from the tail of \mathbf{A} to the tip of \mathbf{C} . Or, you could first put the tail of \mathbf{C} on the head of \mathbf{B} , then put the tail of \mathbf{B} on the head of \mathbf{A} and draw the resultant vector as before. The resulting vector is the same in both cases, so

$$(\mathbf{A} + \mathbf{B}) + \mathbf{C} = \mathbf{A} + (\mathbf{B} + \mathbf{C}).$$

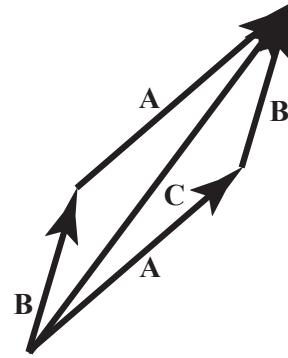
In addition to being commutative, we say that vector addition is also *associative*. We can add as many vectors in this way as we want.



What if we want to *subtract* a vector from another? Suppose we start with our vectors \mathbf{A} and \mathbf{B} , as before. To subtract \mathbf{B} from \mathbf{A} , all we do is *flip* \mathbf{B} around 180° , making $-\mathbf{B}$, and then add it to \mathbf{A} , as in the figure to the left. Then we draw a resultant vector, \mathbf{C} , from the tail of \mathbf{A} to the tip of \mathbf{B} . So, this means that

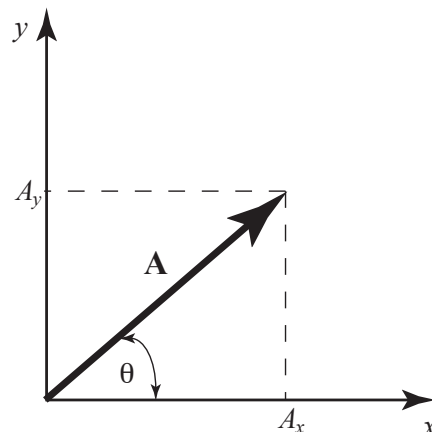
$$\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}).$$

To subtract \mathbf{A} from \mathbf{B} is done in the same way, now flipping \mathbf{A} around, and putting the tail of \mathbf{A} on the tip of \mathbf{B} . Then you draw the resultant vector.



2.2 Vectors in Component Form

Representing vectors as arrows is a nice geometric picture that works well for certain applications, but sometimes we may want more information than the arrow can provide. For example, suppose we want to know how far the arrow \mathbf{A} goes to the right, and how far it goes up. In order to do this, we need a *coordinate system*. Draw a usual two-dimensional Cartesian coordinate system with x and y axes, as seen in the figure to the right, with the tail of the vector set at the origin, being careful not to change the direction of the vector. The vector makes an angle θ with respect to the x axis.



The tip of the arrow stretches a certain distance along the x axis, which is labeled as A_x . This is the x -component of the vector \mathbf{A} . Similarly, the tip of the arrow has a y -component, A_y . So, instead of representing the vector as an arrow with a certain direction, we can specify it by saying how far it goes along the x and y axes. How do we find these?

Suppose that the magnitude of the vector is written as A , without either the boldface type, or the arrow over the top. The magnitude of a vector is often written as $A = |\mathbf{A}|$. So, the length of the arrow is A . From the geometry of the figure and Pythagorus' theorem, we see that

$$A = \sqrt{A_x^2 + A_y^2},$$

where we take the plus sign for the square root since it's a length. So, if we know the components of the vector, we immediately know it's length, which is always positive. We can also find the angle that the vector makes with the x axis. Again, from the geometry we see that the tangent of the angle, $\tan \theta = \frac{A_y}{A_x}$, and so

$$\theta = \tan^{-1} \left(\frac{A_y}{A_x} \right).$$

So, knowing the components of the vector, we can rewrite it as a magnitude and direction.

Suppose we go the other way; suppose we know the length and direction of \mathbf{A} and want the components. Again, going back to the figure, and recalling that the length of \mathbf{A} is A , then

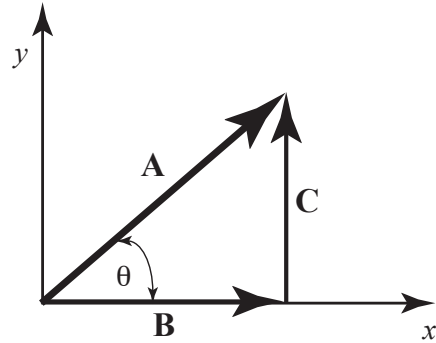
$$\begin{aligned} A_x &= A \cos \theta \\ A_y &= A \sin \theta. \end{aligned}$$

Notice that if we divide A_y by A_x we get $\tan \theta$, as before. So, we now have the components of the vector, and we can express it in terms of these components. There are several ways to do this. One way is to give the components in parenthesis, $\mathbf{A} = (A_x, A_y)$. However, there is a more common way that we will use.

We've expressed our vector by saying how far along the x axis it goes (the x component), and how far along the y axis it goes (the y component). Consider the figure to the right. Using the ideas of adding vectors that we discussed before, we can write

$$\mathbf{A} = \mathbf{B} + \mathbf{C},$$

where the vector \mathbf{B} is *entirely* along the x axis, and \mathbf{C} is *entirely* along the y axis. In other words, we write the vector \mathbf{A} as the sum of two vectors - one along x and the other along y . Choosing our vectors to lie along these axes is a very convenient *basis* to use.



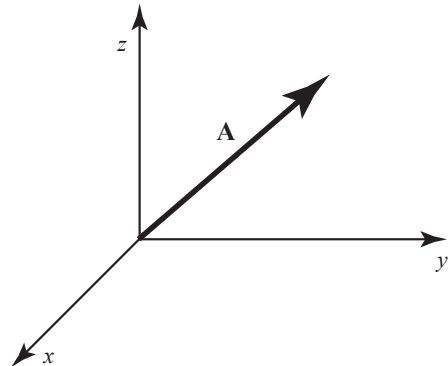
Now, what are the vectors \mathbf{B} and \mathbf{C} ? The length of \mathbf{B} is just the x component of \mathbf{A} , $|\mathbf{B}| = B = A_x$. Similarly, the length of \mathbf{C} is just the y component, $|\mathbf{C}| = C = A_y$. Because we can always describe a vector as a magnitude and a direction, we can say that \mathbf{B} has a length A_x , and points along the x axis. Now, a convenient notation for a vector that points along the x axis is to give it's length, multiplied by a *unit vector*, \hat{i} , which has a length of 1, by definition. So, suppose that the vector \mathbf{B} has a length 3 and points along the x axis. It can then be written as $\mathbf{B} = 3\hat{i}$. More generally for us, $\mathbf{B} = A_x\hat{i}$, since the magnitude of \mathbf{B} is A_x .

What about the vector \mathbf{C} ? We can express \mathbf{C} as a vector of length A_y , pointing along the y axis. We can just do the same thing as before, writing $\mathbf{C} = A_y\hat{j}$, where \hat{j} is another unit vector, this time pointing along the y axis. So, since $\mathbf{A} = \mathbf{B} + \mathbf{C}$, we can plug in $\mathbf{B} = A_x\hat{i}$, and $\mathbf{C} = A_y\hat{j}$, to finally find

$$\mathbf{A} = A_x\hat{i} + A_y\hat{j}.$$

This is a complete specification of the vector \mathbf{A} in terms of it's components. Sometimes the unit vectors are denoted slightly differently, writing \hat{x} and \hat{y} instead of \hat{i} and \hat{j} , but it's just a notation. We'll stick with \hat{i} and \hat{j} .

So far, all the vectors that we have discussed had only two components, A_x and A_y , corresponding to a two-dimensional vector. This is useful for drawing vectors on a 2D piece of paper, but we really live in three dimensions, x , y , and z . For example, suppose you walk 4 meters straight ahead, turn left and walk 3 meters, and then climb 2 meters up a ladder. How far away from your starting point are you? The vector that would connect your final spot to your starting places now moves along all three axes, as seen in the figure to the right.



It's very straightforward to generalize the vector to three dimensions. All we have to do

is to add a piece that tells us how far the vector moves along the z axis. If the component along the z axis is A_z , and if we denote the unit vector along this axis as \hat{k} , then we can write the vector as

$$\mathbf{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k},$$

where the length of the vector is now

$$|\mathbf{A}| = A = \sqrt{A_x^2 + A_y^2 + A_z^2}.$$

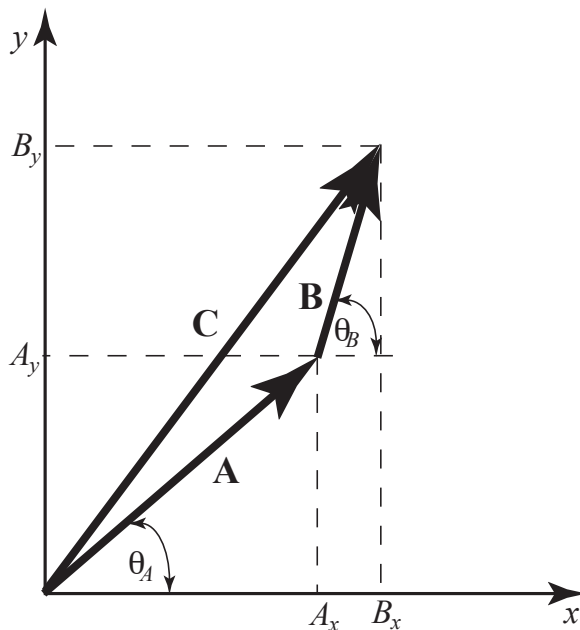
Notice that if the vector *doesn't* go into the z axis (meaning that $A_z = 0$), then we get back the old 2D result. For what follows we'll limit ourselves to the 2D case, but the expressions generalize straightforwardly.

2.2.1 Adding the Components

We've seen how to add two vectors graphically by adding the arrows. How can we do it using the components? Consider the figure to the right, where we add the vectors \mathbf{A} and \mathbf{B} to get the resultant vector \mathbf{C} . Using the ideas discussed above, we can write the resultant vector as

$$\mathbf{C} = C_x \hat{i} + C_y \hat{j}.$$

How do the components C_x and C_y compare to the components of \mathbf{A} and \mathbf{B} ? From the figure, we can see that C_x stretches along the x axis, and is given by the sum of the A_x component, which stretches from the origin to the point labeled A_x , plus the B_x component, which stretches from the point A_x to the point B_x . So, $C_x = A_x + B_x$. It's easy to see that $C_y = A_y + B_y$ in the same way.



So, it seems clear that to add two vectors, we just add the components. Let's see how this works algebraically instead of geometrically. We know that we can write $\mathbf{C} = C_x \hat{i} + C_y \hat{j}$, in general, while $\mathbf{A} = A_x \hat{i} + A_y \hat{j}$, and $\mathbf{B} = B_x \hat{i} + B_y \hat{j}$. So, $\mathbf{C} = \mathbf{A} + \mathbf{B}$, gives

$$\begin{aligned} C_x \hat{i} + C_y \hat{j} &= (A_x \hat{i} + A_y \hat{j}) + (B_x \hat{i} + B_y \hat{j}) \\ &= (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j}, \end{aligned}$$

after collecting the terms. Now, the unit vector \hat{i} points along the x axis, while \hat{j} points along y . These two vectors point in perpendicular directions, and are independent of each other (how far the vector stretches along x has nothing to do with how far it goes along y). This means that we can compare the left- and right-hand sides of our expression and match the coefficients of the unit vectors. This means that

$$\begin{aligned} C_x &= A_x + B_x \\ C_y &= A_y + B_y. \end{aligned}$$

So, it really is true that we just add the components to add the vectors. What if we want to subtract two vectors, in terms of the components? Recall that $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B})$. Now, $-\mathbf{B} = -(B_x\hat{i} + B_y\hat{j}) = -B_x\hat{i} - B_y\hat{j}$. So, adding the negative of \mathbf{B} means that we just subtract the components,

$$\mathbf{A} - \mathbf{B} = (A_x - B_x)\hat{i} + (A_y - B_y)\hat{j}.$$

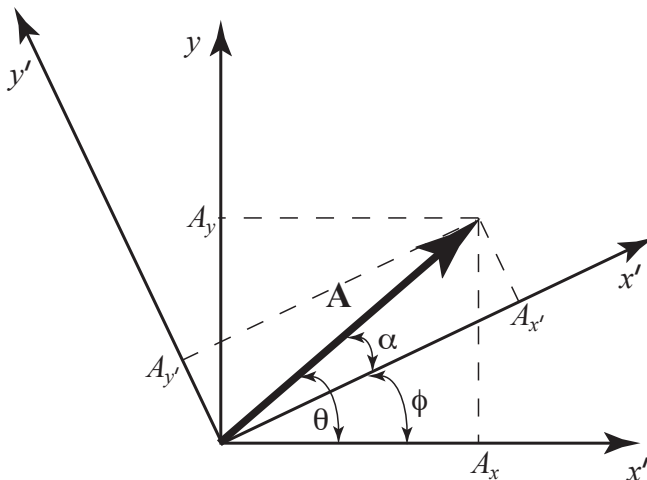
Thus, completely generally we have

$$\mathbf{A} \pm \mathbf{B} = (A_x \pm B_x)\hat{i} + (A_y \pm B_y)\hat{j} + (A_z \pm B_z)\hat{k}.$$

2.3 Rotating the Coordinate Axes

We began our discussion of vectors by describing them as arrows pointing along a certain direction. This is the purely geometrical description of the vector that doesn't need any sort of coordinate system - we drew the arrows without any axes. Later we introduced a coordinate axis so that we could figure out the components of the vector. We decided to draw a particular axis such that the vector made an angle θ with respect to the x axis. This gave us components, $A_x = A \cos \theta$, and $A_y = A \sin \theta$. But - we just introduced that coordinate system, without any real motivation. How do we know it's right? What if someone else used a different coordinate system, such that the vector made a different angle with respect to the x axis? Wouldn't the components be different?

Let's try to figure this out. Suppose that we look at our coordinate system, and then imagine that someone else uses a *different* coordinate system that is *rotated* relative to ours by an angle ϕ . This means that the new x and y axes, denoted with a prime, x' and y' , are rotated by ϕ . We superimpose the two coordinate systems as in the figure to the right. Notice that we haven't touched the vector at all! Now, we still see the same components for the vector, $A_x = A \cos \theta$, and $A_y = A \sin \theta$. What does the other person see?



The other person sees new components, $A_{x'}$ and $A_{y'}$, because they are using new axes. If the vector makes an angle α with respect to the x' axis, then the other person finds $A_{x'} = A \cos \alpha$, and $A_{y'} = A \sin \alpha$. Now, from the diagram, $\alpha = \theta - \phi$, and so

$$\begin{aligned} A_{x'} &= A \cos(\theta - \phi) \\ A_{y'} &= A \sin(\theta - \phi). \end{aligned}$$

So, unless $\phi = 0$ (meaning we use the same coordinates), we don't see the same components, in general. Isn't this a problem? After all, we never touched the vector, itself - only the coordinate axes. Did we change the vector by changing the components?

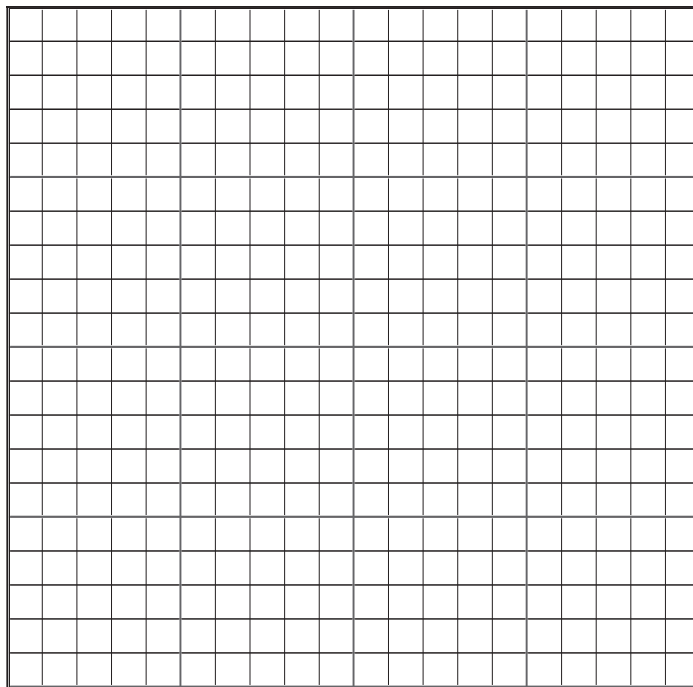
It turns out that this is not a problem. Although the components have, indeed, changed, the vector still points in the same direction (although the angle that we use to define that direction has changed). All we have done is to turn our head a little bit and look at it at an angle. The real, physical vector hasn't changed. This is the important idea: even though the components that we use to describe the vector depend on our choice of coordinate system, any real physical properties of the system don't!

One obvious physical property of the vector is its length. A real arrow that's 30 cm long will always be 30 cm long, no matter how you twist or turn it. For us, the length is $\sqrt{A_x^2 + A_y^2} = \sqrt{A^2 \cos^2 \theta + A^2 \sin^2 \theta} = A$. For our friend, the length is $\sqrt{A_{x'}^2 + A_{y'}^2} = \sqrt{A^2 \cos^2 (\theta - \phi) + A^2 \sin^2 (\theta - \phi)} = A$, and so we agree! The length of the arrow is a real, physical thing, and cannot change, no matter how we rotate (or slide around) the vector.

3 Some Prelab Questions

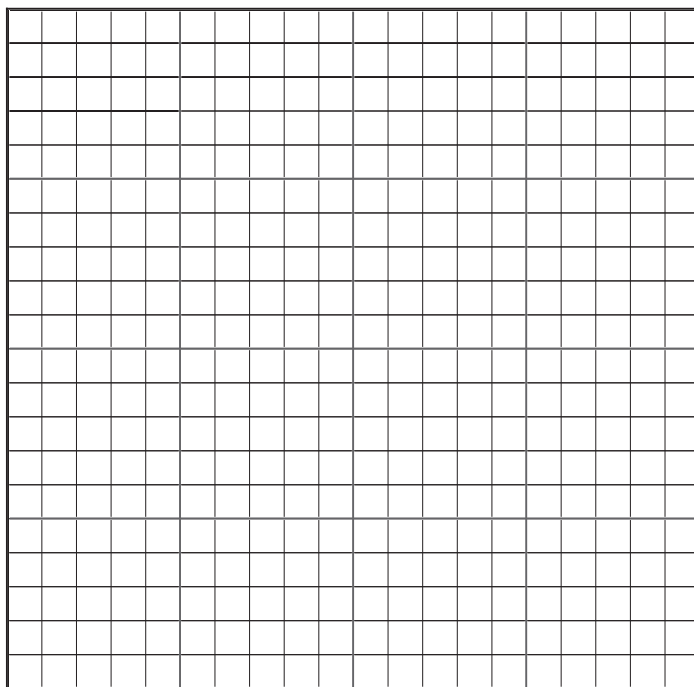
1. On the graph to the right, draw a coordinate axis, and also the following vectors:

- (a) $3\hat{i}$
- (b) $4\hat{j}$
- (c) $3\hat{i} + 4\hat{j}$
- (d) $5\hat{i} + \hat{j}$



2. What are the angles of each of the vectors, as measured from the x axis, in question 1?

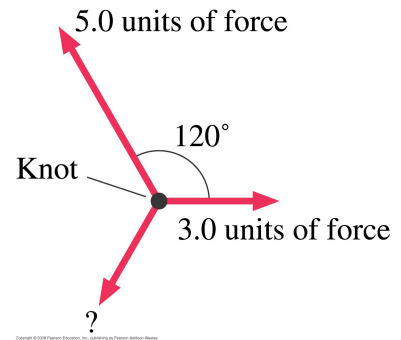
3. On the graph to the right, draw a coordinate axis, and add $\mathbf{A} = 4\hat{i} + 3\hat{j}$ and $\mathbf{B} = 2\hat{i} + 5\hat{j}$ graphically (i.e., add the arrows). What is the equation of the resultant vector? What angle does it make with respect to the x axis?



4. Suppose you go out for a walk. You walk straight ahead for 100 meters. Then you turn to your left and walk for another 50 meters. Finally you turn to your right and walk another 25 meters. How far are you from where you started? At what angle are you, with respect to the direction you started out walking? Do this problem using components.

5. Suppose you don't know two vectors \mathbf{A} and \mathbf{B} , but you do know $\mathbf{C} = \mathbf{A} + \mathbf{B}$, and $\mathbf{D} = \mathbf{A} - \mathbf{B}$. Can you figure out \mathbf{A} and \mathbf{B} ?

6. The figure to the right shows three ropes tied together in a knot. One of your friends pulls on a rope with 3.0 units of force and another pulls on a second rope with 5.0 units of force. How hard, and in what direction, must you pull on the third rope to keep the knot from moving?



4 The Experiment

Now that we have become experts in vector manipulations, we want to apply these ideas to a real system. One of the most important vectors in physics is the force acting on a system, \mathbf{F} . Newton tells us that if there are several forces acting on a system, \mathbf{F}_1 , \mathbf{F}_2 , etc., then the net force acting on the system is just the (vector) sum of the forces

$$\mathbf{F}_{\text{net}} = \sum_i \mathbf{F}_i.$$

In particular, if the forces all cancel out then the net force on the system is zero. A system with no net force acting on it is said to be in *equilibrium*. Newton also tells us that $\mathbf{F} = m\mathbf{a}$, and so a system in equilibrium will not accelerate. A good way of studying a number of forces acting on a system is through the use of a force table, which we'll discuss now.

4.1 The Force Table Background

A force table is seen in the figure to the right. The table consists of a circular table with markings ticking off 360° . Pulleys can be attached to the edge of the table, allowing weights to be suspended from strings. Several weights can be hung from a central ring, leading to several forces, \mathbf{F}_i , acting on the ring. When the forces are balanced such that the net force is zero, then the ring is at equilibrium. It will be the purpose of this experiment to determine the conditions for equilibrium for several different mass configurations.



4.2 Experimental Setup

Materials Needed

- Force Table, including central ring and retaining pin
- Three Pulleys
- String
- Three Hanging mass rack with masses
- Protractor
- Ruler
- Bubble level

Begin by using the bubble level to be sure that the table is level. There are a set of leveling screws on the tripod to adjust the angle of the table. Place the retaining pin in the center of the force table. Attach two pulleys to opposite ends of the table, at angles of 0° and 180° . Next, attach two pieces of string to the central ring, each being long enough that they can stretch over the pulley and hang several centimeters below the table. Next, attach the mass racks to the free ends of the strings. Hang the mass racks over each of the pulleys, with the retaining pin inside the ring.

4.3 The Experiment

We'll begin with the simplest case: two hanging masses in one dimension. After that, we'll add another mass, and another dimension.

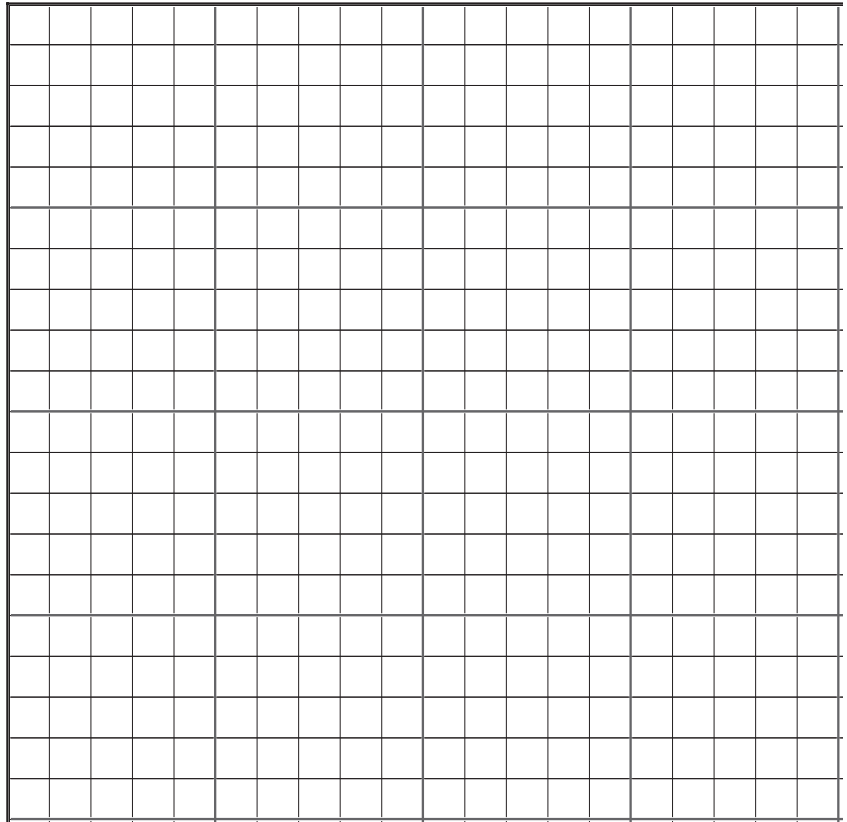
4.3.1 One Dimension

Begin by placing 50 grams on each of the hanging mass racks. Pull the retaining pin out of the table. Does the ring move, or does it stay in one place? Why? If the ring moves, what do you need to do to make it stay?

If the ring moves, then make your adjustments such that it stays fixed. What is the magnitude of the force on each of the masses? The same two forces are acting on the central ring. What are the directions of the forces acting on the rings (i.e., what are the angles)? Finally, express each force as a vector in component form, taking $\theta = 0$ to be the x axis. Record each of these values in the data table below.

1D Trial 1 Results			
Force	Magnitude (N)	Direction (θ)	Vector
\mathbf{F}_1			
\mathbf{F}_2			

On the graph to the right, draw a coordinate axis. Draw in the two force vectors. Show by adding the vectors graphically that the net force on the central ring is zero. Verify this algebraically by showing that the sum of the components is also zero.

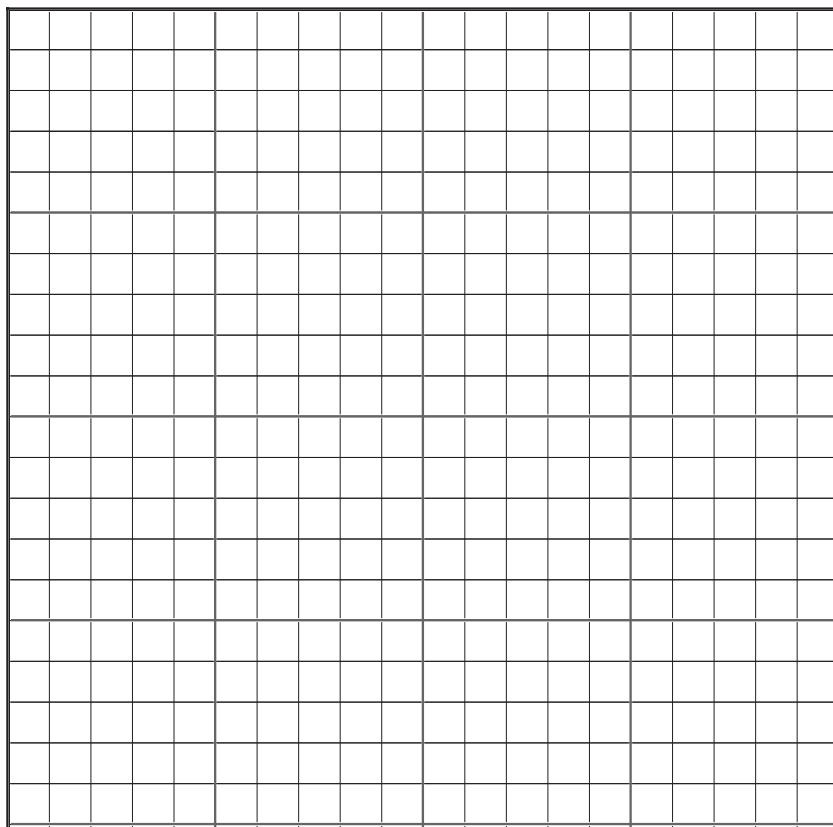


Now, notice that you can slide the ring back and forth along the x axis, placing it at any distance, and the ring will stay in place. This means that every point along the x axis is an equilibrium point. Explain why this is the case.

Now we're going to change the axes. Remove the two pulleys, and place them at angles of 90° and 270° , respectively. Keeping the x axis along $\theta = 0$, we've now placed the forces along the y axis. Repeat the earlier analysis with the new forces.

1D Trial 2 Results			
Force	Magnitude (N)	Direction (θ)	Vector
\mathbf{F}_1			
\mathbf{F}_2			

On the graph to the right, draw a coordinate axis. Draw in the two force vectors. Show by adding the vectors graphically that the net force on the central ring is zero. Verify this algebraically by showing that the sum of the components is also zero.



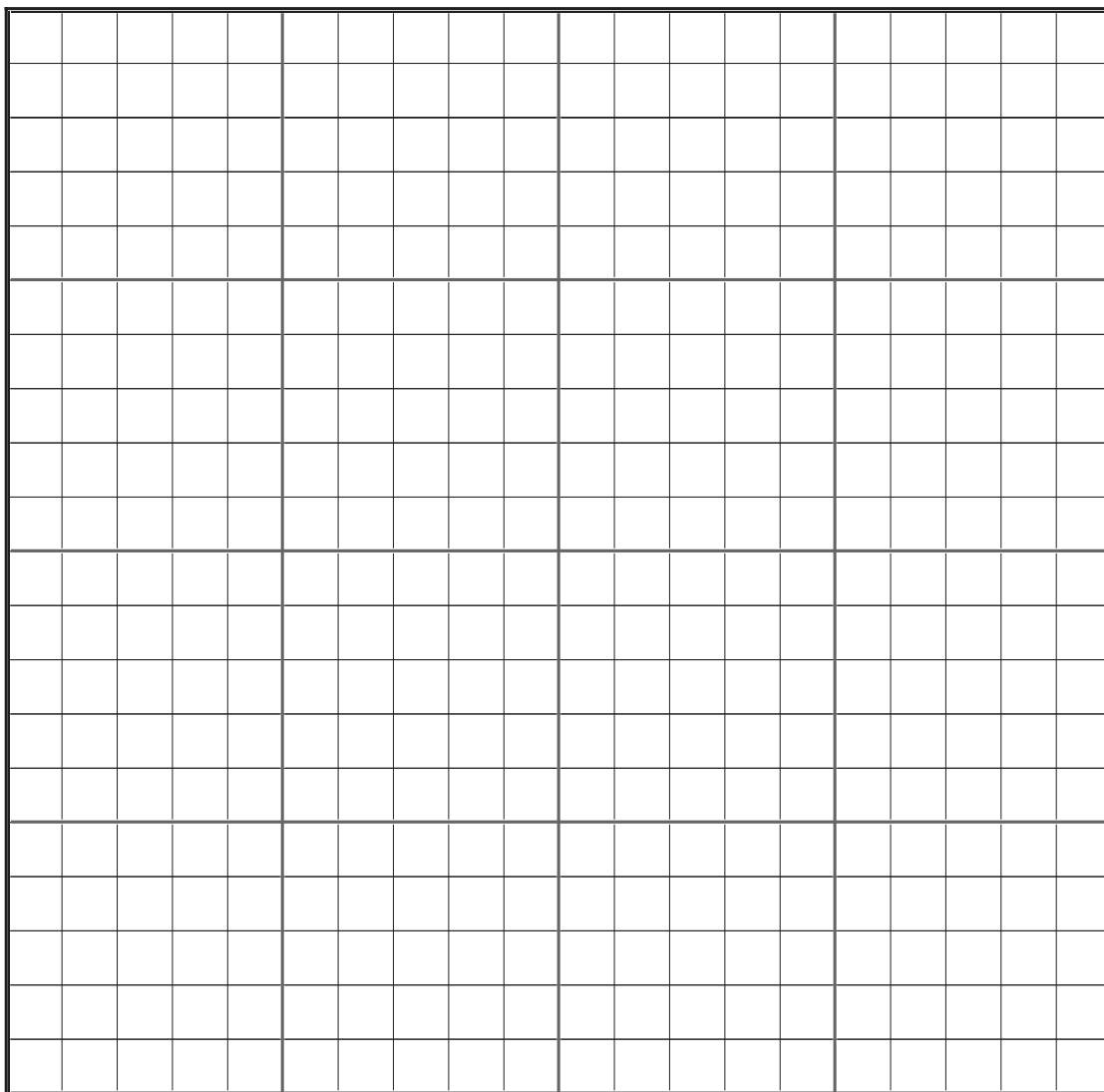
We can still slide the ring back and forth, only now along the y axis. Now every point along the y axis is an equilibrium point. What does this tell you?

4.3.2 Two Dimensions

Now we'll add a slight complication. Place the pulleys at angles $\theta = 45^\circ$ and 225° . If the x axis is still defined by $\theta = 0$, then the forces now have *both* x and y components. This now makes the system *two-dimensional*. Repeat the experiment with these new values.

2D Trial 1 Results			
Force	Magnitude (N)	Direction (θ)	Vector
\mathbf{F}_1			
\mathbf{F}_2			

On the graph below, draw a coordinate axis. Draw in the two force vectors. Show by adding the vectors graphically that the net force on the central ring is zero. Verify this algebraically by showing that the sum of the components is also zero.

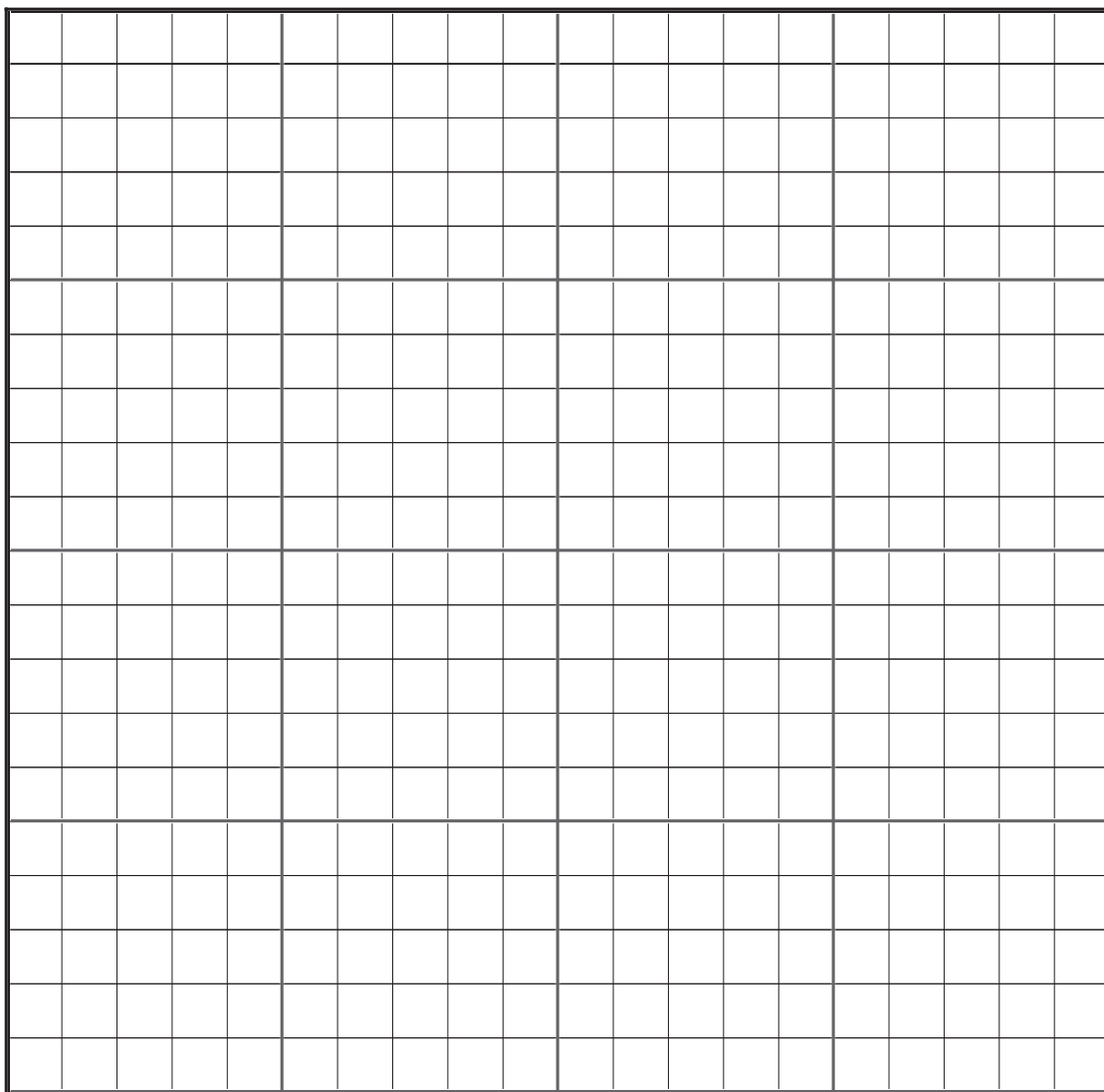


Yet again, we can still slide the ring back and forth, only now along the new axis. What does this tell you?

Now, let's add another pulley. Attach a third string to the ring, and the other end to the third mass rack. Now, place two pulleys at angles $\theta = 0^\circ$, and $\theta = 120^\circ$. Replace the retaining pin through the ring in the center of the table. Hang two mass racks over the pulleys, and place 100 grams on each of them. Place a third pulley at some angle at which you expect the system to balance (roughly what angle do you expect this to be?). Now, place 100 grams on the final mass rack, and thread it's string over the third pulley. In general, the system won't balance for the random angle that you have chosen. Slide the pulley around until the ring balances in the center. When it does, remove the retaining pin. For the angle that you have found, enter your values in the following table.

2D Trial 2 Results			
Force	Magnitude (N)	Direction (θ)	Vector
F_1			
F_2			
F_3			

On the graph below, draw a coordinate axis. Draw in the three force vectors. Show by adding the vectors graphically that the net force on the central ring is zero. Verify this algebraically by showing that the sum of the components is also zero.

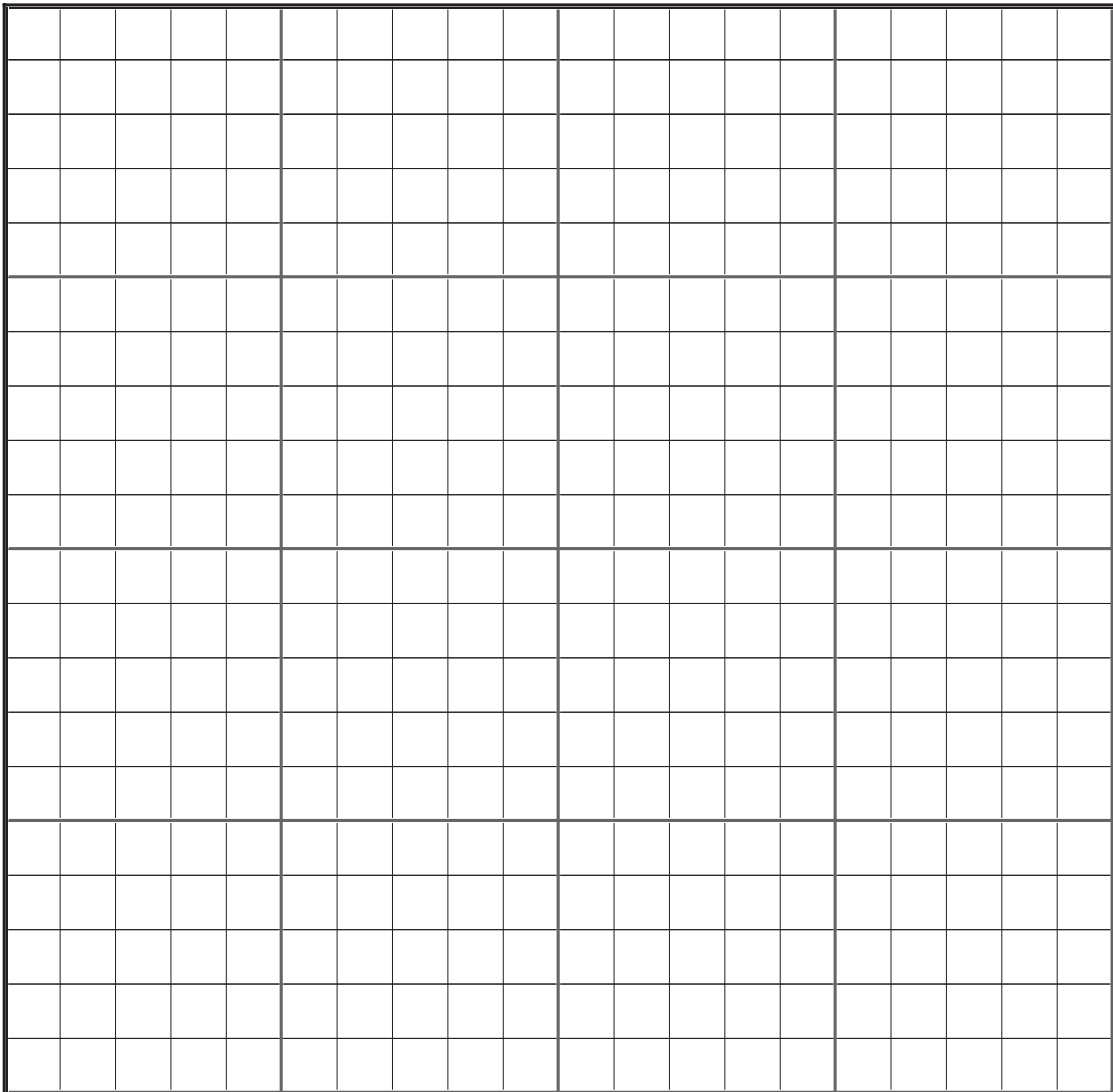


In the one-dimensional case, we could slide the ring back and forth, and have it stay where we set it. Can we do the same in this case? Why or why not?

So, translation of the origin doesn't work in the two-dimensional case. What about rotation of the coordinate system? Rotate *all* of your pulleys *clockwise* by 120° , but *keep the relative angles between the pulleys fixed*. Does the system still balance? Repeat the earlier experiment for your new setup.

2D Trial 3 Results			
Force	Magnitude (N)	Direction (θ)	Vector
\mathbf{F}_1			
\mathbf{F}_2			
\mathbf{F}_3			

On the graph below, draw a coordinate axis. Draw in the three force vectors. Show by adding the vectors graphically that the net force on the central ring is zero. Verify this algebraically by showing that the sum of the components is also zero.



Finally, place 100 grams on a rack over a pulley at $\theta = 45^\circ$. Next, place 200 grams on a rack over a pulley at $\theta = 150^\circ$. How much mass, and at what angle, do you need to balance the forces? Record your prediction in the data table below.

2D Trial 4 Results			
Force	Magnitude (N)	Direction (θ)	Vector
\mathbf{F}_1			
\mathbf{F}_2			
\mathbf{F}_3 (theoretical)			
\mathbf{F}_4 (experimental)			

Now, perform the experiment and measure the angle and mass at which the system balances. Record the experimental value in the data table. Do your results agree? If not, what could be throwing off your results (i.e., what are some possible sources of error)?

5 Some Last Questions

1. A field mouse trying to escape a hawk runs east for 5.0 m, darts southeast for 3.0 m, then drops 1.0 m straight down a hole into its burrow. What is the magnitude of the net displacement of the mouse?
2. A flock of ducks is trying to migrate south for the winter, but they keep being blown off course by a wind blowing from the west at 6.0 m/s. A wise elder duck finally realizes that the solution is to fly at an angle relative to the wind. If the ducks can fly at 8.0 m/s relative to the air, what direction should they head in order to move directly south?
3. Show that the components of a vector in a coordinate system rotated by an angle ϕ can be given in terms of the original components as

$$\begin{aligned}A_{x'} &= A_x \cos \phi + A_y \sin \phi \\A_{y'} &= -A_x \sin \phi + A_y \cos \phi.\end{aligned}$$