Notes 7-4: Similarity in Right Triangles

A right triangle is a triangle with _____________________________.

An ____________ of a triangle is the ____________________ segment from a ___________ to the side of that vertex.

When you draw the __________________ to the _____________________ of a right triangle, you form triangles.

Example: Identify the following in right triangle XYZ.

1. the hypotenuse

2. the segments of the hypotenuse

3. the altitude to the hypotenuse

Example: Write a similarity statement relating the three triangles in each diagram.

geometric mean (noun) jee uh MEH trik meen

Definition: For any two positive numbers a and b, the geometric mean of a and b is the positive number x such that \( \frac{a}{x} = \frac{x}{b} \).

Example: The geometric mean of 4 and 10 is the value of x in \( \frac{4}{x} = \frac{x}{10} \), or \( x = 2\sqrt{10} \).
Example: Find the geometric mean of each pair of numbers

$4$ and $9$

\[
\frac{4}{x} = \frac{x^2}{9} \rightarrow x^2 = 36 \rightarrow x = 6
\]

$14$ and $12$

$6$ and $500$

We can use the geometric mean to write proportions using lengths in right triangles without thinking through the similar triangles.

Example: Solve for $x$ and $y$.

\[x \quad 5 \]

\[y \quad 5 \]

\[y \quad x \quad 50 \]

\[100 \]

\[40 \quad 60 \]

\[21 \quad y \quad x \]

\[9 \]
Notes 7-4: Similarity in Right Triangles

A right triangle is a triangle with a right angle.

An **altitude** of a triangle is the perpendicular segment from a **vertex** to the side of that vertex.

When you draw the **altitude** to the **hypotenuse** of a right triangle, you form 3 similar triangles.

**Example:** Identify the following in right triangle XYZ.

1. the hypotenuse $\overline{XY}$
2. the segments of the hypotenuse $\overline{XR} + \overline{RY}$
3. the altitude to the hypotenuse $\overline{ZR}$

**Example:** Write a similarity statement relating the three triangles in each diagram.

$\Delta RSO \sim \Delta QST \sim \Delta KQT$

$\Delta NOP \sim \Delta NQO \sim \Delta OQP$

**geometric mean** (noun) jee uh MÖH trik meen

**Definition:** For any two positive numbers $a$ and $b$, the geometric mean of $a$ and $b$ is the positive number $x$ such that $\frac{a}{x} = \frac{x}{b}$. $x^2 = a \cdot b$ so $x = \sqrt{a \cdot b}$

**Example:** The geometric mean of 4 and 10 is the value of $x$ in $\frac{4}{x} = \frac{x}{10}$ or $x = 2\sqrt{10}$. 
Example: Find the geometric mean of each pair of numbers

4 and 9
\[
\frac{4}{x} = \frac{9}{x} \implies x^2 = 36 \implies x = \sqrt{36} = 6
\]

14 and 12
\[
\frac{14}{x} = \frac{x}{12} \implies x^2 = 168 \implies x = \sqrt{168} \approx 12.99
\]

6 and 500
\[
\frac{6}{x} = \frac{x}{500} \implies x^2 = 3000 \implies x = \sqrt{3000} \approx 54.77
\]

We can use the geometric mean to write proportions using lengths in right triangles without thinking through the similar triangles.

Example: Solve for x and y.

\[
\frac{SL}{SL} = \frac{LL}{LL} \implies \frac{5}{y} = \frac{y}{5} \implies y^2 = 25 \implies y = \sqrt{25} \quad y = 5
\]

\[
\frac{10}{x} = \frac{x}{5} \implies x^2 = 50 \implies x = \sqrt{50} \quad x \approx 7.07
\]

Example: Find the geometric mean of each pair of numbers.

We can use the geometric mean to write proportions using lengths in right triangles without thinking through the similar triangles.